BICS Away Day
Theme D Presentation

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Theme D

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Domain Decomposition Methods for Elliptic PDEs

- typical equation : diffusion with variable coefficients

$$-\nabla \cdot (\alpha \nabla u) = f$$

- applications : flow in porous media, materials with microstructures
- goal : efficient w.r.t. problem size, coefficients
- method
  - discretise $\longrightarrow$ large system of equations, ill-conditioned
  - domain decomposition : divide problem into many small subproblems
  - two-level method : additionally solve coarse problem

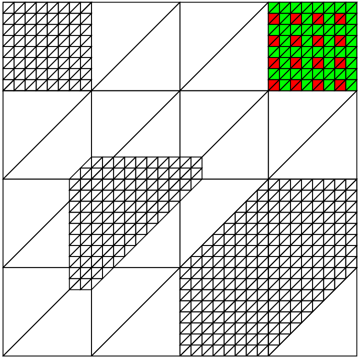
Research Topics

- staggered grids
- deflation : work on links between additive and hybrid Schwarz and deflation
  implementation, conferences (rs), paper (rs,igg)
- last BICS away day : choice of boundary conditions for coarse basis
- new development : energy minimising coarse spaces

Robust Coarse Spaces

- solve local problems to construct coarse space basis
- boundary conditions on coarse grid s.t.
  partitition of unity
- alternative : simple zero boundary conditions on
  overlapping supports,
  implicitely impose partition of unity constraint
- equivalent to constrained energy minimisation
  (studied by multigrid community)
- need to solve system of same size as original
- but very structured, can be done efficiently

Example: Fine Scale Binary Medium



$\alpha = 10^6$

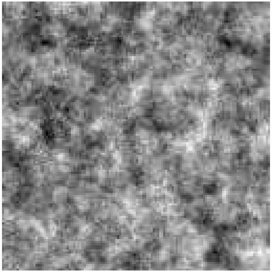
n_s	n	m	one	lin	linbc	oscbc	erg
4	8	32	24	34	34	24	25
8	8	64	40	59	62	27	27
16	8	128	77	112	115	26	26
32	8	256	154	219	240	26	26

$n_s = 32,\ n = 8,\ m = 256$

α	one	lin	linbc	oscbc	erg
10^0	129	22	22	22	22
10^2	132	81	52	23	23
10^4	132	218	218	25	26
10^6	154	219	240	26	26

Example: Gausian Random Field

mean 0, variance σ^2,
correlation length λ



$\lambda = 5h$

σ^2	one	lin	linbc	oscbc	erg
0	67	22	22	22	23
2	162	44	40	36	35
4	226	65	55	46	44
8	377	121	94	65	62
12	531	199	146	86	81
16	662	304	213	108	103
20	819	440	297	133	126

$\alpha = \exp \mathrm{Gaussian}$

Robustness of Coarse Basis Construction

binary medium

α	A	D	E	C
10^0	53	43	18	10
10^2	70	108	56	10
10^4	71	119	134	9
10^6	71	37	200^+	9

n_s	n	m	A	D	E	C
4	8	32	18	33	167	10
8	8	64	31	37	200^+	10
16	8	128	52	38	200^+	10
32	8	256	71	37	200^+	9

Gaussian field

σ^2	A	D	E	C
0	38	44	18	10
2	96	94	37	13
4	138	164	55	14
8	200	200^+	96	15
12	200^+	200^+	152	16
16	200^+	200^+	199	16
20	200^+	200^+	200^+	17

Future

- finish papers
- convergence analysis
- choice of supports (aggregation method)
  visit Eero Vainikko (Tartu)
- tensor problems
- non-symmetric problems (e.g. convection)
- start collaboration with cjb : adaptive methods

Presentations

- Numerical Analysis Seminars (dd 15/12/06, convection 16/2)
- Computational Science Workshop (Bath, 10/1)
- Multigrid Conference (Copper Mountain, CO, 16/3)
- UTexas Seminar (Austin, TX, 25/3)
- Numerical Analysis Conference (Dundee, 26/6)
- NAMMAC (Bath, 5/9)

Many discussions at Copper, Austin, NAMMAC.

Internal and External Links

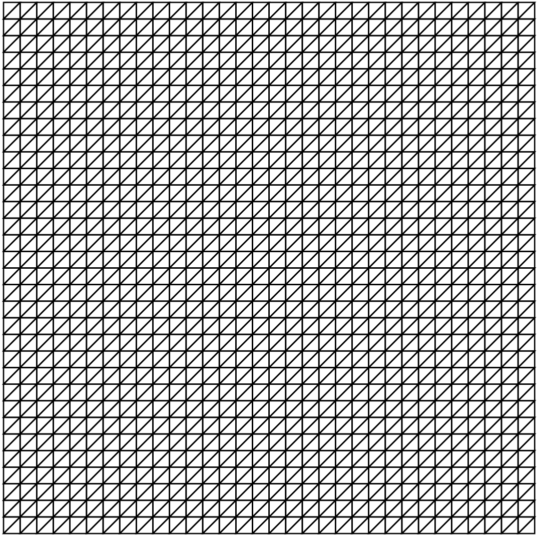
- **Jeremy Campbell** MSc project 'Modelling the Injection of CO_2 into Deep Saline Aquifers'
  - supervisors : **Steven Benbow (Quintessa)**, **Jonathan Evans**, jvl
  - system of coupled diffusion equations
  - several non-linear constitutive relations
  - cylindrically symmetric model
  - self-similar solution
- **Sean Buckeridge** PhD project 'Numerical Solution of Weather and Climate Models'
  - supervisors : rs/jvl and **Mike Cullen (Met Office)**
  - integrate multigrid methods in Met Office code
  - spherical geometry, anisotropy, varying coefficients
  - parallelisation

External Links

- **Eero Vainikko (Tartu)** domain decomposition code
- **Sabine Le Borne (TTU)** convection
- **Mary Wheeler & co (Austin)** multiscale methods
- **Paul Godden (ICR)** modelling ultrasound treatment, help with modelling, numerics, programming, linux
- **Tom Hou, Jay Chu (Caltech)** multiscale methods
- **Ian Sloan (UNSW)** approximation on sphere

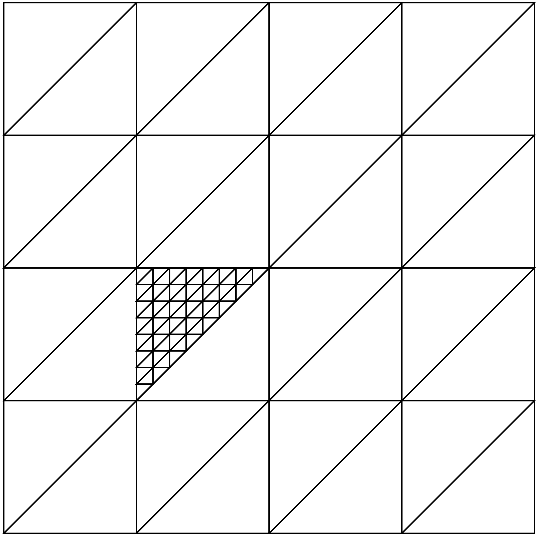
Two-Level Overlapping Schwarz Method

- fine grid



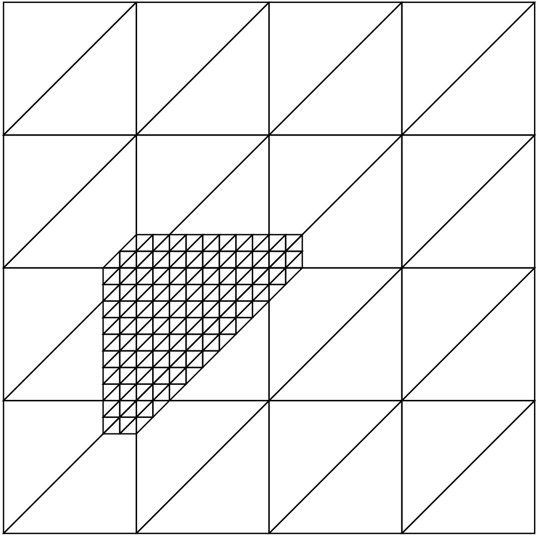
Two-Level Overlapping Schwarz Method

- fine grid
- subdomains



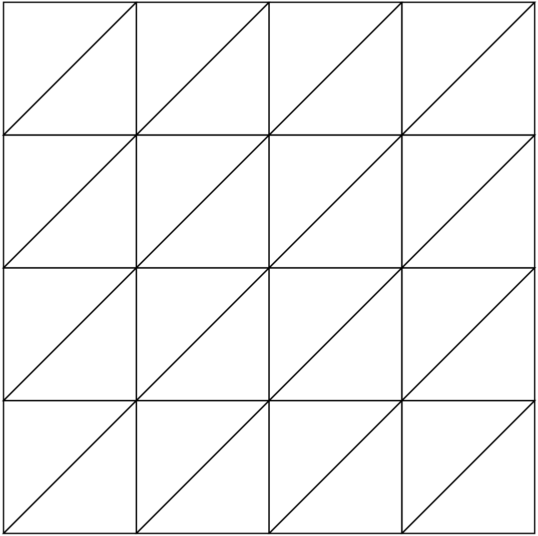
Two-Level Overlapping Schwarz Method

- fine grid
- subdomains
- overlap



Two-Level Overlapping Schwarz Method

- fine grid
- subdomains
- overlap
- coarse grid



Two-Level Overlapping Schwarz Method

- fine grid
- subdomains
- overlap
- coarse grid
- coarse basis