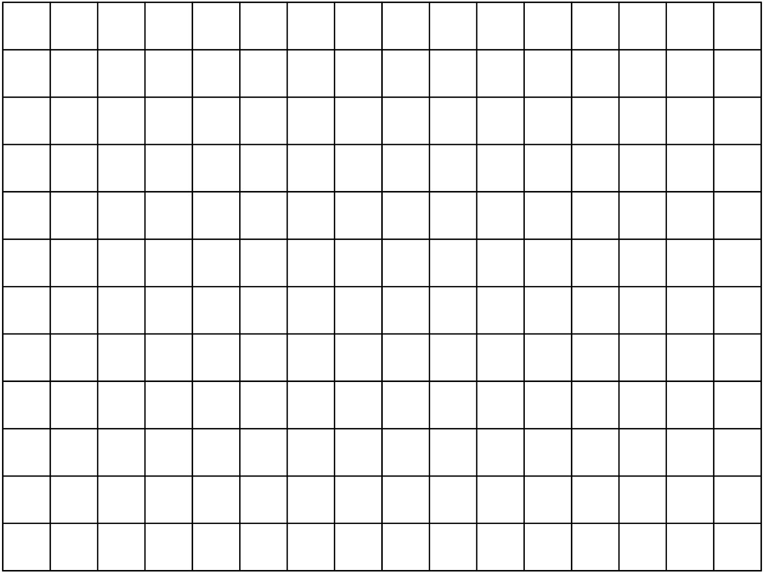
Elliptic PDEs with Highly Varying Coefficients

- model problem : elliptic equation

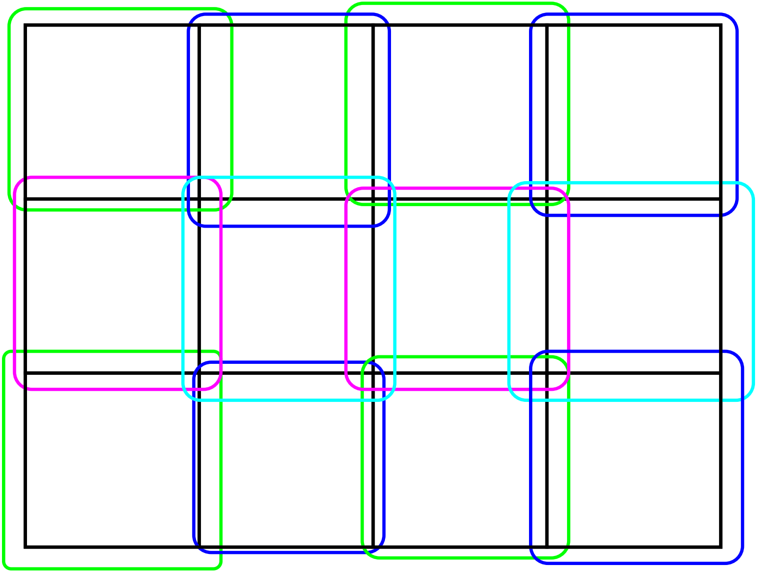
$$\operatorname{div}(\alpha \operatorname{grad} u) = f$$

- highly varying $\alpha(x, y)$
- applications :
  flow in porous media, microstructures in materials
- discretisation : FD, FV, FE
- large system of equations : iterative methods
- preconditioner : domain decomposition
- aim : robust w.r.t. variation in $\alpha(x, y)$
- test cases : binary media, random media

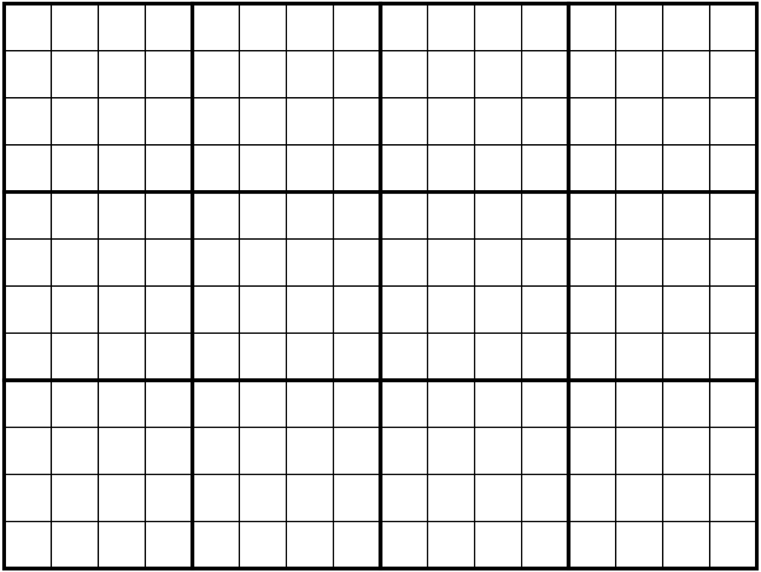
Grid



Overlapping Subdomains

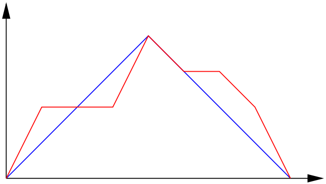


Coarse Grid



Coarse Basis Functions

- subdomain problems in parallel
- only subdomains : slow
- add coarse grid : robust method
- incorporate coefficients
- choice of coarse grid basis is very important
- 1D example
- 2D : boundary conditions of subproblems



New Developments

- implementation to get familiarised with methods
- multiplicative combination of local and coarse solves
- deflation with coarse basis functions
- staggered grids
- more test cases

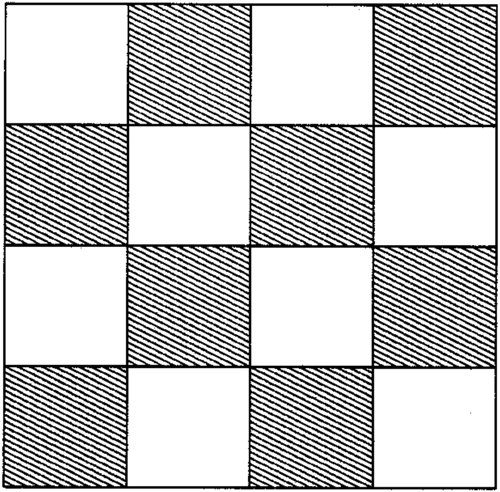
Outlook

- choice of boundary conditions
- started study of aggregation method and code
- concept of strong connections
- non-symmetric problems (e.g. convection)
- approximation using multiscale basis functions

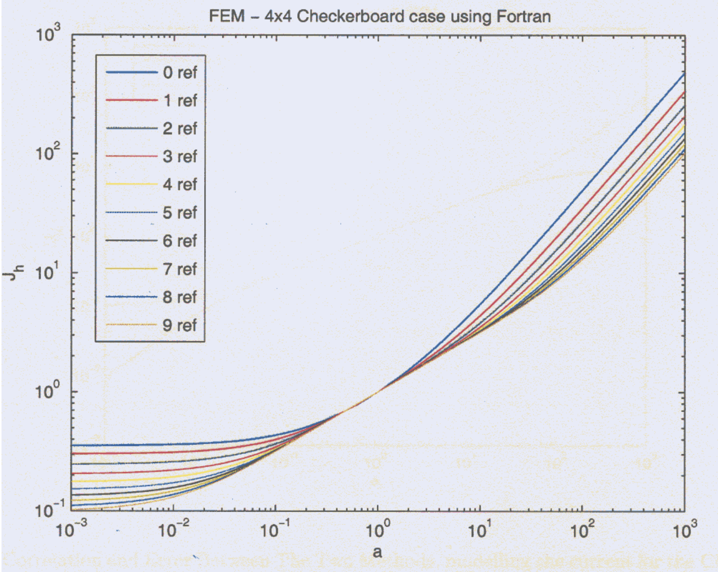
Materials with Microstructures

- same equation
- properties of material at small scale are unknown
- assume random field
- goal : estimate bulk properties
- discretise using FE or FV
- if elements same scale as coefficient patches,
  then similar to network models
- MSc project Sean Buckeridge : study discretisation

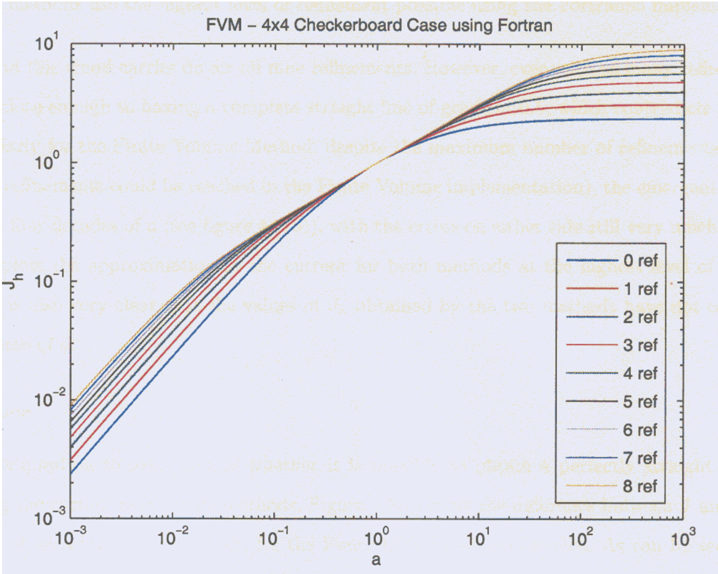
Test Case: Checkerboard



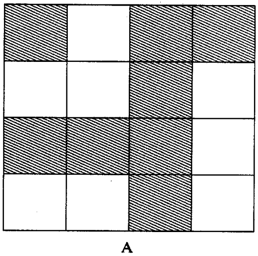
Finite Elements

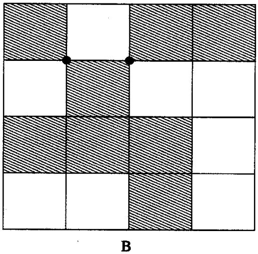


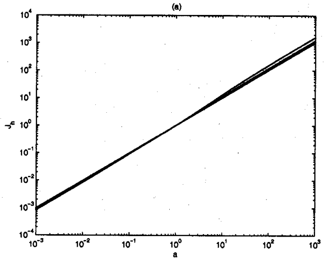
Finite Volumes

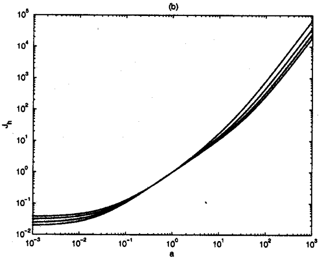


Cross Points

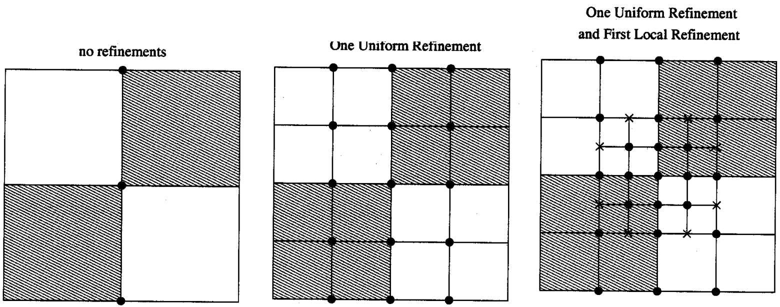




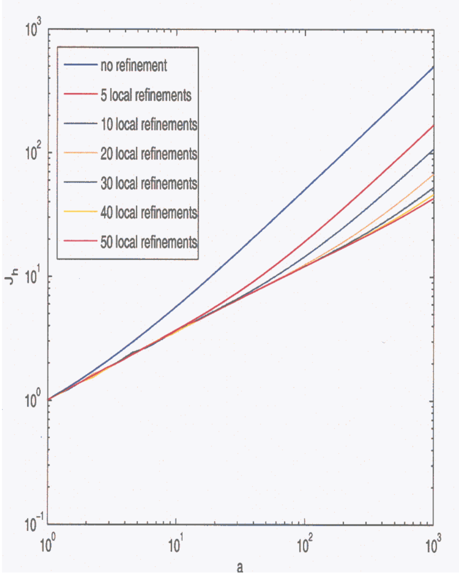




Local Refinement



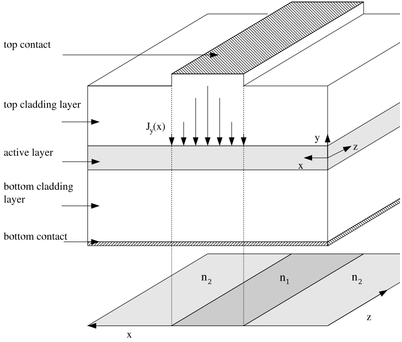
Local Refinement Results



Laser Physics

collaboration with Federica Causa (E & E Eng.)
MSc project Peter Way

- absorption : photon bumps electron to higher energy level
- spontaneous emission : electron falls to lower energy level and emits photon
- stimulated emission : emitted photon has same properties as passing photon



Coupled Carrier Diffusion and Wave Propagation

- model
  - Maxwell's equation for electromagnetic field
  - diffusion for carriers
- simplifications:
  harmonic (no t), slab (no y), slow variation (z)
  - wave equation: 2 complex quantities f^+ and f^-

$$f_{xx}^{\pm} \mp ipf_z^{\pm} + \tilde{\epsilon}(N)f^{\pm} = 0$$

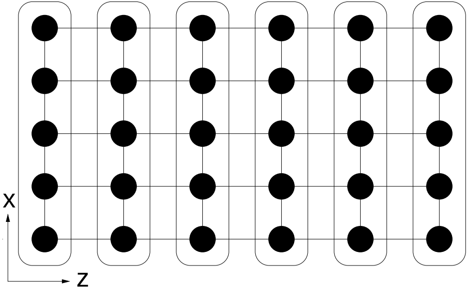
  - carrier: 1 real quantity N

$$DN_{xx} + R(f^+, f^-, N) + \tilde{J} = 0$$

- boundary conditions

Simulation

- discretise x
- system of ODEs + algebraic equations
- iterate forward and backward through z
- at each step
  - propagate field f^+ or f^- : linear if carrier given
  - update carrier N : nonlinear



Nonlinear Equations

- fixed point iteration : slow
- Newton iteration : much faster
- total time from hours to minutes or even seconds

Outlook

- Newton on whole system
- exploit block structure of Jacobian matrix
- detect instabilities via bifurcation analysis
- Peter Way will work on this for one more month